COMBINATORIAL AND GEOMETRIC GRAPHS

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In previous lectures we studied properties of subgraphs, such as paths, circuits, spanning trees, and cut-sets, in a given connected graph *G*. In this chapter we shall subject the entire graph *G* to the following important question: Is it possible to draw *G* in a plane without its edges crossing over?

This question of planarity is of great significance from a theoretical point of view. In addition, planarity and other related concepts are useful in many practical situations. For instance in the design of a printed-circuit board, the electrical engineer must know if he can make the required connections without an extra layer of insulation.

The solution to the puzzle of three utilities, posed in Chapter 1, requires the knowledge of whether or not the corresponding graph can be drawn in a plane.

But before we attempt to draw a graph in a plane, let us examine the meaning of "drawing" a graph.

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As mentioned in Chapter 1, a graph exists as an abstract object, devoid of any geometric connotation of its ability of being drawn in a three-dimensional Euclidean space. For example, an abstract graph G_1 can be defined as

$$G_1 = (V, E, \Psi)$$

where the set *V* consists of the five objects named *a*, *b*, *c*, *d*, and *e*, that is,

$$V = \{a, b, c, d, e\},$$

and the set *E* consists of seven objects (none of which is in set *V*) named 1, 2, 3, 4, 5, 6, and 7, that is,

$$E = \{1, 2, 3, 4, 5, 6, 7\},$$

and the relationship between the two sets is defined by the mapping Ψ, which consists of

$$\Psi = \begin{bmatrix} 1 \longrightarrow (a, c) \\ 2 \longrightarrow (c, d) \\ 3 \longrightarrow (a, d) \\ 4 \longrightarrow (a, b). \\ 5 \longrightarrow (b, d) \\ 6 \longrightarrow (d, e) \\ 7 \longrightarrow (b, e) \end{bmatrix}$$

Here, the symbol $1 \rightarrow (a, c)$ says that object 1 from set *E* is mapped onto the (unordered) pair (a, c) of objects from set *V*.

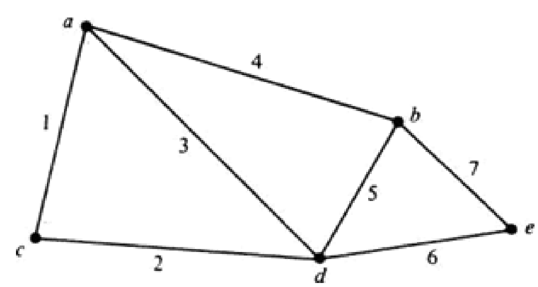

Fig. 2-13 Unicursal graph.

Now it so happens that this combinatorial abstract object G_1 can also be *represented by* means of a geometric figure. In fact, the sketch in Fig. 2-13 is one such geometric representation of this graph. Moreover, it is also true that any graph can be represented by means of such a configuration in three-dimensional Euclidean space.

It is important to realize that what is sketched in Fig. 2-13 is merely one (out of infinitely many) representation of the graph G_1 and *not the graph* G_1 *itself*. We could have, for instance, twisted some of the edges or could have placed *e* within the triangle a, *d, b* and thereby obtained a different figure representing G_1.

References

- Deo, N, Graph theory with applications to Engineering and Computer Science, PHI.
- Gary Chartrand and Ping Zhang, Introduction to Graph Theory, TMH.
- Robin J. Wilson, Introduction to Graph Theory, Pearson Education.