ALL CUTSETS IN A GRAPH

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4-2. SOME PROPERTIES OF A CUT-SET

Consider a spanning tree *T* in a connected graph *G* and an arbitrary cutset *S* in G. Is it possible for *S* not to have any edge in common with *T*? The answer is *no*. Otherwise, removal of the cut-set *S* from *G* would not disconnect the graph. Therefore,

THEOREM 4-1

Every cut-set in a connected graph G must contain at least one branch of every spanning tree of G.

Will the converse also be true? In other words, will any minimal set of edges containing at least one branch of every spanning tree be a cut-set? The answer is *yes*.

THEOREM 4-2

In a connected graph *G*, any minimal set of edges containing at least one branch of every spanning tree of *G* is a cut-set.

THEOREM 4-3

Every circuit has an even number of edges in common with any cut-set.

4-3. ALL CUTSETS IN A GRAPH

In Section 4-1 it was shown how cutsets are used to identify weak spots in a communication net. For this purpose we list all cutsets of the corresponding graph, and find which ones have the smallest number of edges. It must also have become apparent to you that even in a simple example, such as in Fig. 4-1, there is a large number of cutsets, and we must have a systematic method of generating all relevant cutsets.

In the case of circuits, we solved a similar problem by the simple technique of finding a set of ***fundamental circuits*** and then realizing that other circuits in a graph are just *combinations* of two or more fundamental circuits. We shall follow a similar strategy here. Just as a spanning tree is essential for defining a set of fundamental circuits, so is a spanning tree essential for a set of *fundamental cutsets*. It will be beneficial for us to look for the parallelism between circuits and cutsets.

Fundamental CutSets:

Consider a spanning tree T of a connected graph G. Take any branch b in T. Since {b} is a cut-set in T, {b} partitions all vertices of T into two disjoint sets—one at each end of b.

Consider the same partition of vertices in G, and the cut set S in G that corresponds to this partition. Cutset S will contain only one branch b of T, and the rest (if any) of the edges in S are chords with respect to T. Such a cut-set S containing exactly one branch of a tree T is called a **fundamental cut-set** with respect to T. Sometimes a fundamental cut-set is also called a **basic cut-set**.

In Fig. 4-3, a spanning tree T (in heavy lines) and all five of the fundamental cutsets with respect to T are shown (broken lines "cutting" through each cut-set).

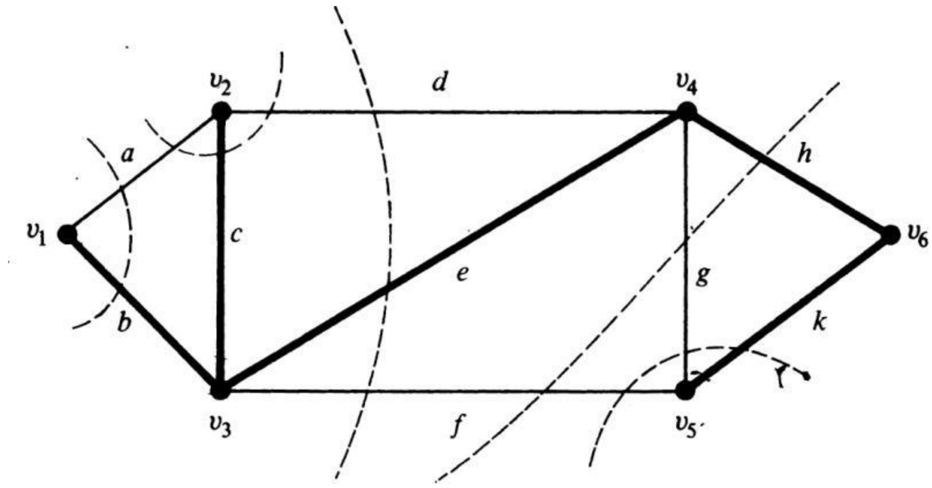


Fig. 4-3 Fundamental cutsets of a graph.

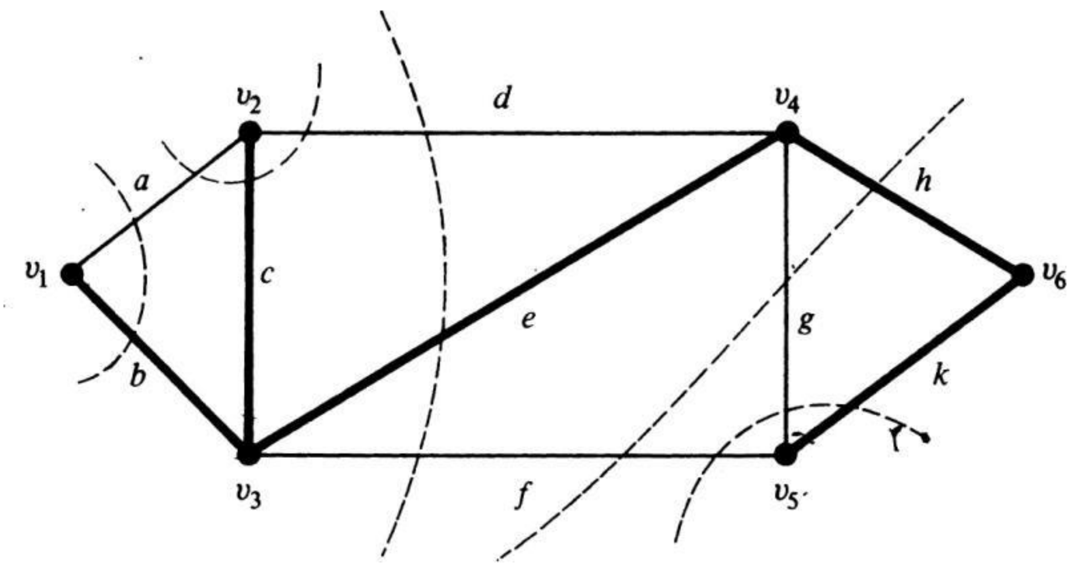
Just as **every chord** of a spanning tree defines a ***unique* fundamental circuit**, **every branch** of a spanning tree defines **a *unique* fundamental cut-set**. It must also be kept in mind that the term fundamental cut-set (like the term fundamental circuit) has meaning only with respect to a *given* spanning tree.

Now we shall show how other cutsets of a graph can be obtained from a given set of cutsets.

THEOREM 4-4

The ring sum of any two cutsets in a graph is either a third cut-set or an edge-disjoint union of cutsets.

Example: In Fig. 4-3 let us consider ring sums of the following three pairs of cutsets.



$$\{d, e, f\} \oplus \{f, g, h\} = \{d, e, g, h\}, \quad \text{another cut-set,}$$

$$\{a, b\} \oplus \{b, c, e, f\} = \{a, c, e, f\}, \quad \text{another cut-set,}$$

$$\{d, e, g, h\} \oplus \{f, g, k\} = \{d, e, f, h, k\}$$

$$= \{d, e, f\} \cup \{h, k\}, \text{ an edge-disjoint union of cut-sets.} \quad \blacksquare$$

References

- Deo, N, Graph theory with applications to Engineering and Computer Science, PHI.
- Gary Chartrand and Ping Zhang, Introduction to Graph Theory, TMH.
- Robin J. Wilson, Introduction to Graph Theory, Pearson Education.