Finite Fourier Transform

Unit III Part IV (Integral Transforms)

Dr.Rachana Pathak
Assistant Professor
Department of Applied Science and Humanities,
Faculty of Engineering and Technology, University of Lucknow

October 2020

Disclaimer: The e-content is exclusively meant for academic purposes and for enhancing teaching and learning.Any other use for economic/commercial purpose is strictly prohibited. The users of the content shall not distribute,disseminate or share it with anyone else and its use is restricted to advancement of individual knowledge.The information provided in this e-content is authentic and best as per my knowledge.

Finite Fourier Transforms

The finite Fourier sine transform of $F(x), 0 < x < l$ is defined as

$$f_s(p) = \int_0^l F(x) \sin\frac{p\pi x}{l} dx; p \in I$$

Similarly, the finite Fourier cosine transform of $F(x), 0 < x < l$ is defined as

$$f_c(p) = \int_0^l F(x) \cos\frac{p\pi x}{l} dx; p \in I$$

For simplicity, we choose π as the upper limit of integration

$$f_s(p) = \int_0^\pi F(x) \sin px dx$$

$$f_c(p) = \int_0^\pi F(x) \cos px dx$$

Inverse Finite Fourier Transforms

Inversion formulae are given as follows:

1. **When upper limit is π**

   For sine transform:

   $$F(x) = \frac{2}{\pi} \sum_{p=1}^{\infty} f_s(p) \sin px$$

   For cosine transform:

   $$F(x) = \frac{1}{\pi} f_c(0) + \frac{2}{\pi} \sum_{p=1}^{\infty} f_c(p) \cos px \text{ where } f_c(0) \text{ stands for } \int_0^{\pi} F(x)\, dx$$

2. **When upper limit is l**

   For sine transform:

   $$F(x) = \frac{2}{l} \sum_{p=1}^{\infty} f_s(p) \sin \frac{p\pi x}{l}$$

   For cosine transform:

   $$F(x) = \frac{1}{l} f_c(0) + \frac{2}{l} \sum_{p=1}^{\infty} f_c(p) \cos \frac{p\pi x}{l} \text{ where } f_c(0) \text{ stands for}$$

   $$\int_0^{l} F(x)\, dx.$$

Example

1. Find finite Fourier sine transform of $F(x) = 1 - \frac{x}{\pi}$.

Solution: The finite Fourier sine transform of $F(x)$ is given by,

$$f_s(p) = \int_0^{\pi} \left(1 - \frac{x}{\pi}\right) sinpxdx = \left\{\left(1 - \frac{x}{\pi}\right)\left(-\frac{cospx}{p}\right)\right\}_0^{\pi} - \int_0^{\pi} \left(-\frac{1}{\pi}\right)\left(-\frac{cospx}{p}\right) dx$$

$$= \frac{1}{p} - \frac{1}{\pi p}\left(\frac{sinpx}{p}\right)_0^{\pi} = \frac{1}{p}.$$

2. Find finite Fourier cosine transform of $F(x) = -\frac{cosk(\pi - x)}{ksink\pi}$.

Solution. $f_c(p) = \int_0^{\pi} -\frac{cosk(\pi - x)}{ksink\pi} cospxdx =$

$$-\frac{1}{2ksink\pi} \int_0^{\pi} \left[cos\left\{k(\pi - x) + px\right\} + cos\left\{k(\pi - x) - px\right\}\right] dx$$

$$= -\frac{1}{2ksink\pi}\left[\frac{sin(k\pi - kx + px)}{p - k} - \frac{sin(k\pi - kx - px)}{p + k}\right]_0^{\pi}$$

$$= \frac{1}{2k}\left[\frac{1}{p-k} - \frac{1}{p+k}\right] = \frac{1}{p^2 - k^2}, k \neq 0.1, 2, ...$$

Reference

- A text book of Engineering Mathematics, Vol-III, N.P.Bali, Manish Goel University Science Press.
- Advanced Engineering Mathematics, Fifth Edition, R.K.Jain, S.R.K. Iyenga, Narosa Publication House.
- Higher Engineering Mathematics,B.S.Grewal,Khanna Publishers.
- Higher Engineering Mathematics,B.V.Ramana, McGraw-Hill Publishing Company Limited.
- Advanced Engineering Mathematics, Tenth Edition, Erwin Ereyrzig Wily Publishing.
- Engineering Mathematics, Vol.III and Vol. IV, C.B Gupta, S.R.Singh, McGraw-Hill Publishing Company Limited.

Thank you