FACULTY OF ENGINEERING AND TECHNOLOGY

UNIVERSITY OF LUCKNOW

LUCKNOW

Course: Bachelor of Technology - 3rd Year

Subject: Graph Theory (CS-604)

Topic: FUNDAMENTAL CIRCUITS AND CUTSETS

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4-4. FUNDAMENTAL CIRCUITS AND CUTSETS

Consider a spanning tree T in a given connected graph G. Let c_i be a chord with respect to T, and let the fundamental circuit made by c_i be called Γ, consisting of k branches $b_1, b_2, \ldots, b_k$ in addition to the chord c_i; that is,

Γ = {c_i, b_1, b_2, . . . , b_k) is a fundamental circuit with respect to T.

Every branch of any spanning tree has a fundamental cut-set associated with it.

Let S_1 be the fundamental cut-set associated with b_1, consisting of q chords in addition to the branch b_1; that is,

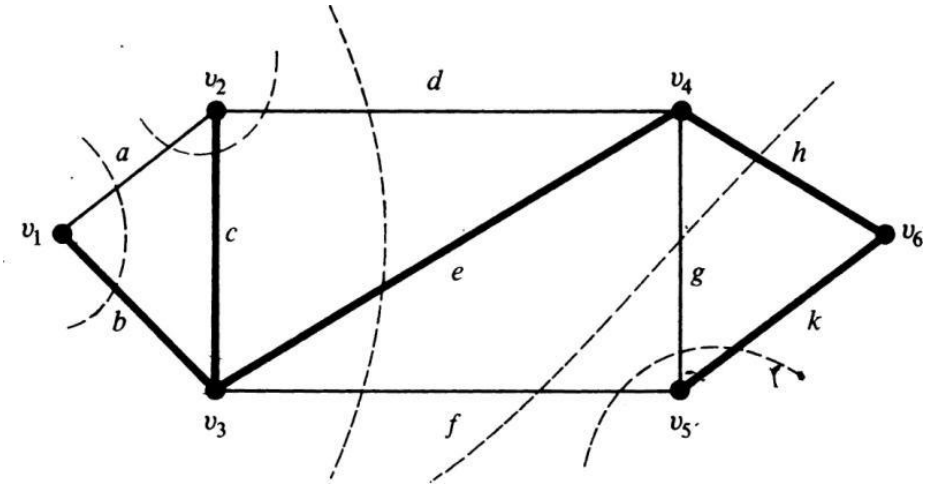
S1 = {bl, cl, c2, . . . , cq} is a fundamental cut-set with respect to T.

Because of Theorem 4-3, there must be an even number of edges common to Γ and S1. Edge b1 is in both Γ and S1, and there is only one other edge in Γ (which is ci) that can possibly also be in S1. Therefore, we must have two edges b1 and ci common to S1 and Γ. Thus the chord ci is one of the chords c1, c2, . . .cq.

Exactly the same argument holds for fundamental cutsets associated with b_2, b_3, . . . , and b_k. Therefore, the chord c_i is contained in every fundamental cut-set associated with branches in Γ.

THEOREM 4-5

With respect to a given spanning tree T, a chord c_j that determines a fundamental circuit Γ occurs in every fundamental cut-set associated with the branches in Γ and in no other.



As an example, consider the spanning tree {*b*, *c*, *e*, *h*, *k*}, shown in heavy lines, in Fig. 4-3. The fundamental circuit made by chord *f* is

{*f*, *e, h, k*}.

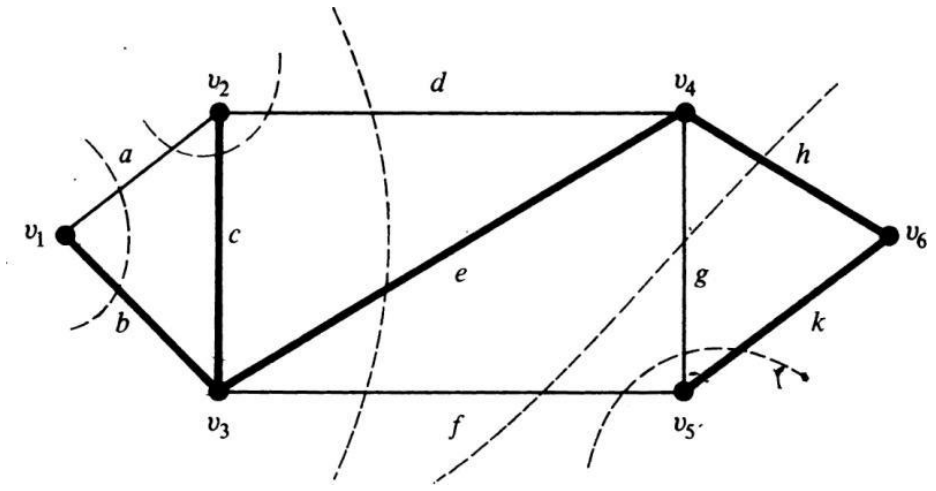
The three fundamental cutsets determined by the three branches e, h, and k are

determined by branch e: {d, e, f},
determined by branch h: {f, g, h},
determined by branch k: {f, g, k},

Chord f occurs in each of these three fundamental cutsets, and there is no other fundamental cut-set that contains f. The converse of Theorem 4-5 is also true.

THEOREM 4-6

With respect to a given spanning tree T, a branch b_j that determines a fundamental cut-set S is contained in every fundamental circuit associated with the chords in S, and in no others.



For illustration,in the graph in Fig. 4-3, consider branch e of spanning tree $\{b, c, e, h, k\}$. The fundamental cut-set determined by e is

$$\{e, d, f\}.$$

The two fundamental circuits determined by chords d and f are

determined by chord d: $\{d, c, e\}$,
determined by chord f: $\{f, e, h, k\}$.

Branch e is contained in both these fundamental circuits, and none of the remaining three fundamental circuits contains branch e.

References

- Deo, N, Graph theory with applications to Engineering and Computer Science, PHI.
- Gary Chartrand and Ping Zhang, Introduction to Graph Theory, TMH.
- Robin J. Wilson, Introduction to Graph Theory, Pearson Education.