

Lecture Notes
for
B.TECH. III yr, Vth Semester
(Electrical Engg.)

Subject Code: EE–502

Subject: Control Systems

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SIGNAL FLOW GRAPHS

Signal flow graph is a graphical representation of algebraic equations.

Each signal flow graph consists of a network in which nodes are connected by directed branches. Each node represents a system variable, and each branch acts as a signal multiplier. The signal flows in the direction indicated by the arrow.

Basic Elements of Signal Flow Graph

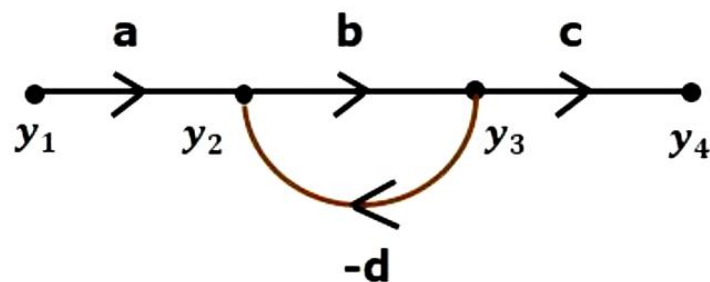
Nodes and branches are the basic elements of signal flow graph.

Node is a point which represents either a variable or a signal. There are three types of nodes — input node, output node and mixed node.

- **Input Node** – It is a node, which has only outgoing branches.
- **Output Node** – It is a node, which has only incoming branches.
- **Mixed Node** – It is a node, which has both incoming and outgoing branches.

Example

Let us consider the following signal flow graph to identify these nodes.



- The **nodes** present in this signal flow graph are **y_1 , y_2 , y_3** and **y_4** .
- **y_1** and **y_4** are the **input node** and **output node** respectively.
- **y_2** and **y_3** are **mixed nodes**.

Branch

Branch is a line segment which joins two nodes. It has both **gain** and **direction**. For example, there are four branches in the above signal flow graph. These branches have **gains** of **a, b, c** and **-d**.

Construction of Signal Flow Graph

Let us construct a signal flow graph by considering the following algebraic equations –

$$y_2 = a_{12}y_1 + a_{42}y_4$$

$$y_3 = a_{23}y_2 + a_{53}y_5$$

$$y_4 = a_{34}y_3$$

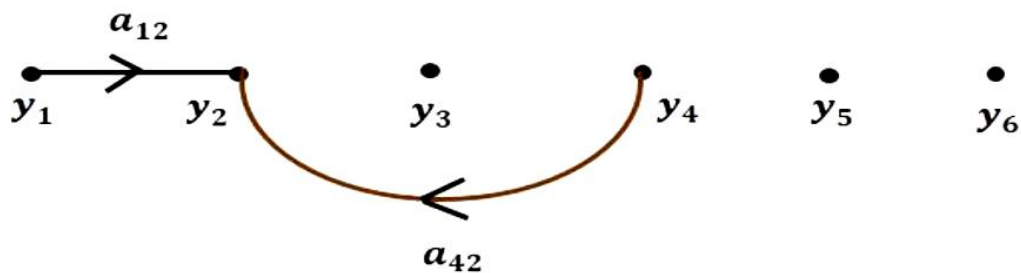
$$y_5 = a_{45}y_4 + a_{35}y_3$$

$$y_6 = a_{56}y_5$$

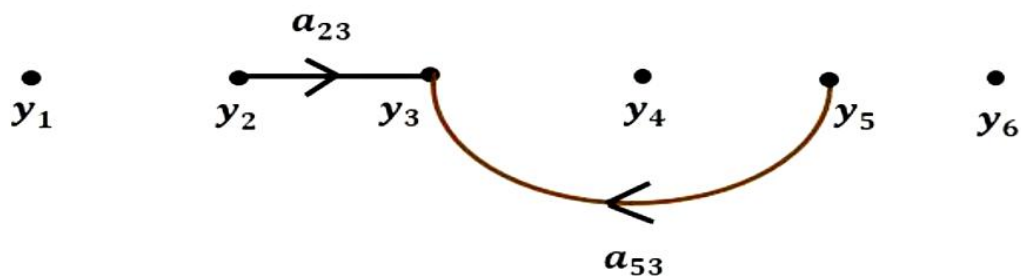
There will be six **nodes** (y_1, y_2, y_3, y_4, y_5 and y_6) and eight **branches** in this signal flow graph. The gains of the branches are $a_{12}, a_{23}, a_{34}, a_{45}, a_{56}, a_{42}, a_{53}$ and a_{35} .

To get the overall signal flow graph, draw the signal flow graph for each equation, then combine all these signal flow graphs and then follow the steps given below –

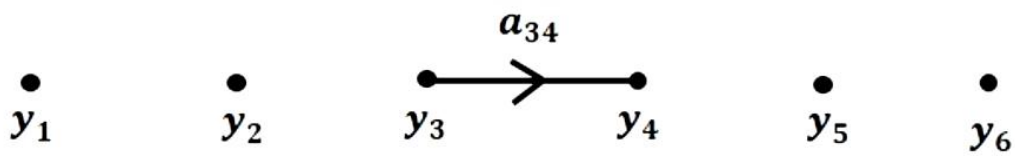
Step 1 – Signal flow graph for $y_2 = a_{12}y_1 + a_{42}y_4$ is shown in the following figure.



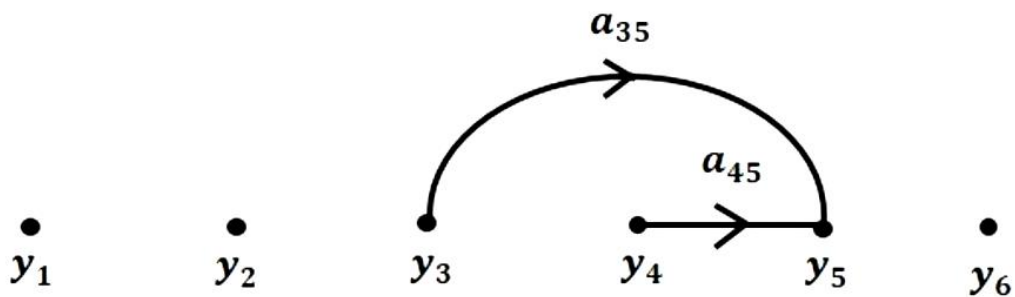
Step 2 – Signal flow graph for $y_3 = a_{23}y_2 + a_{53}y_5$ is shown in the following figure.



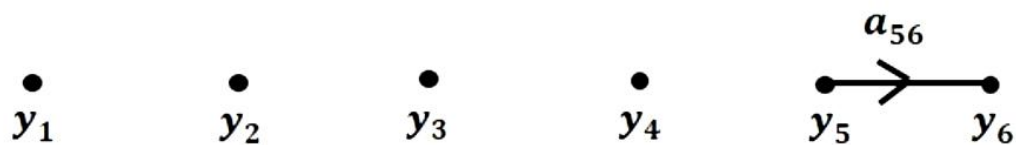
Step 3 – Signal flow graph for $y_4 = a_{34}y_3$ is shown in the following figure.



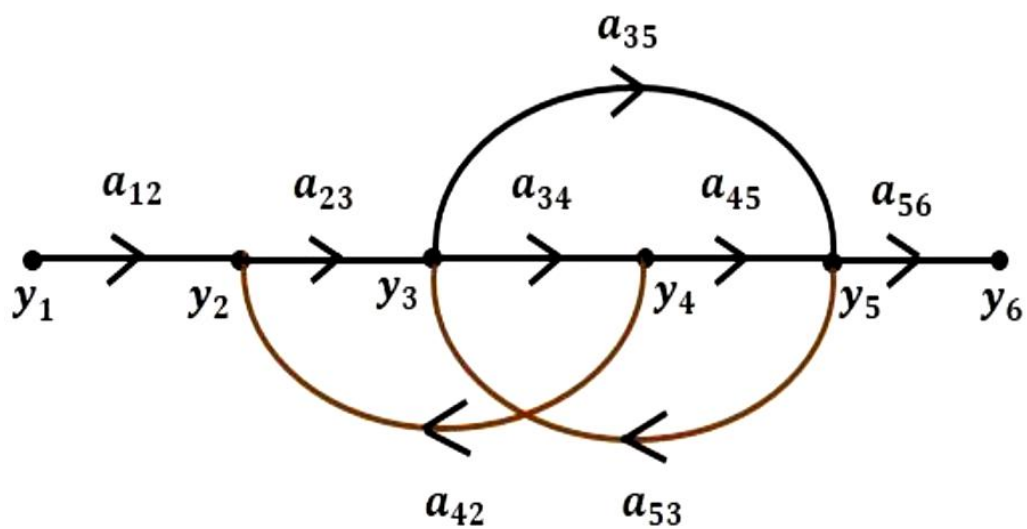
Step 4 – Signal flow graph for $y_5 = a_{45}y_4 + a_{35}y_3$ is shown in the following figure.



Step 5 – Signal flow graph for $y_6 = a_{56}y_5$ is shown in the following figure.



Step 6 – Signal flow graph of overall system is shown in the following figure.



Conversion of Block Diagrams into Signal Flow Graphs

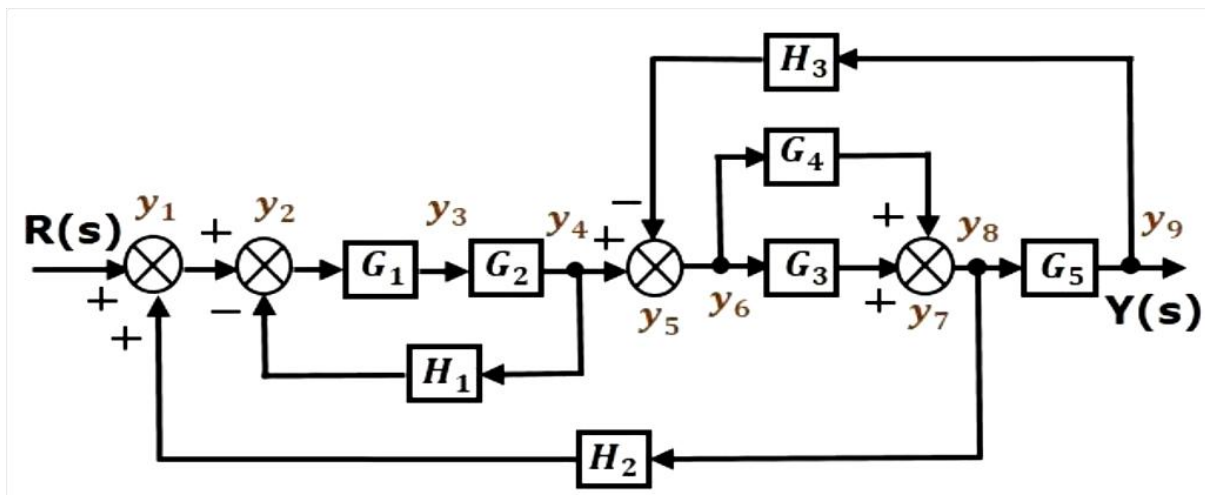
Follow these steps for converting a block diagram into its equivalent signal flow graph.

- Represent all the signals, variables, summing points and take-off points of block diagram as **nodes** in signal flow graph.
- Represent the blocks of block diagram as **branches** in signal flow graph.
- Represent the transfer functions inside the blocks of block diagram as **gains** of the branches in signal flow graph.
- Connect the nodes as per the block diagram. If there is connection between two nodes (but there is no block in between), then represent the gain of the branch as one. **For example**, between summing points, between summing point and takeoff point, between input and summing point, between take-off point and output.

Example

Let us convert the following block diagram into its equivalent signal flow graph.

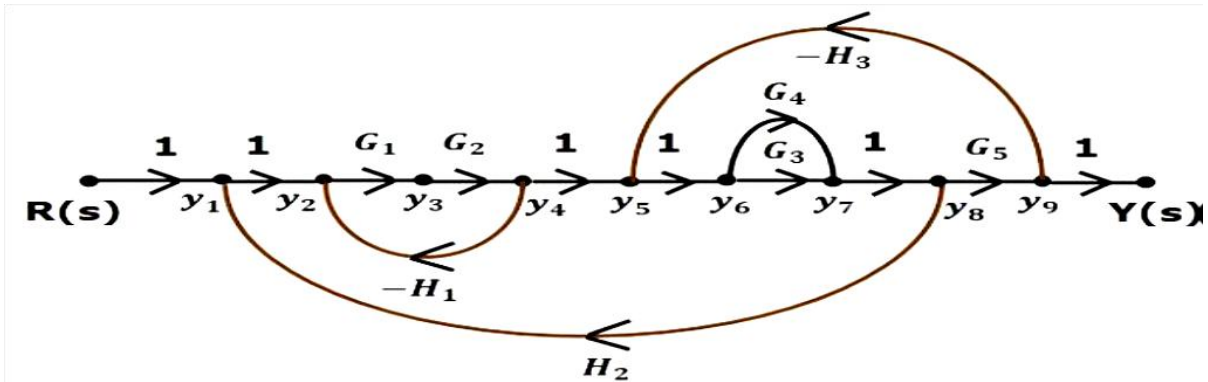
Represent



Represent the input signal $R(s)$ and output signal $C(s)$ of block diagram as input node $R(s)$ and output node $C(s)$ of signal flow graph.

Just for reference, the remaining nodes (y_1 to y_9) are labelled in the block diagram. There are nine nodes other than input and output nodes. That is four nodes for four summing points, four nodes for four take-off points and one node for the variable between blocks G_1 and G_2 .

The following figure shows the equivalent signal flow graph.



Let us now discuss the Mason's Gain Formula. Suppose there are 'N' forward paths in a signal flow graph. The gain between the input and the output nodes of a signal flow graph is nothing but the **transfer function** of the system. It can be calculated by using Mason's gain formula.

Mason's gain formula is

$$T = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^N P_i \Delta_i}{\Delta}$$

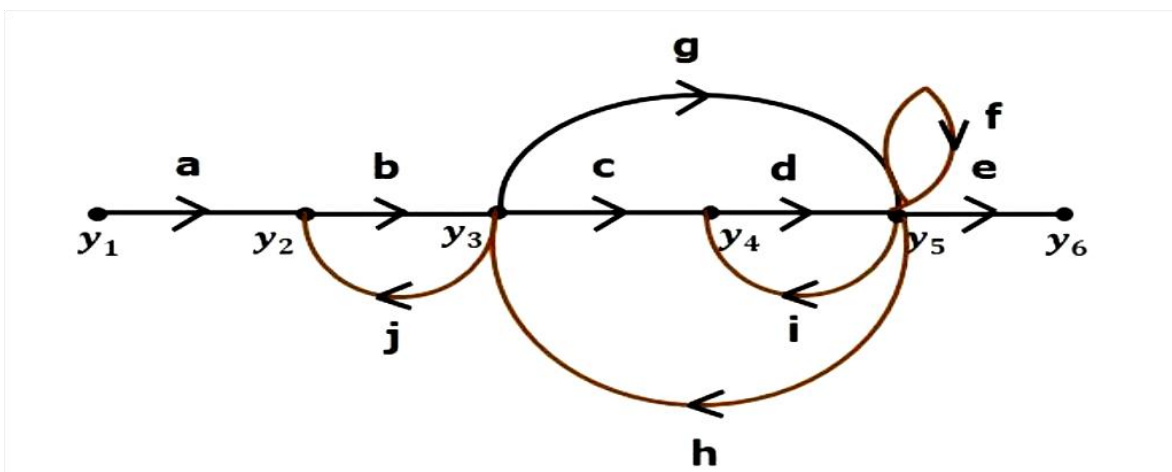
where

- **C(s)** is the output node
- **R(s)** is the input node
- **T** is the transfer function or gain between R(s) and C(s)
- **P_i** is the *i*th forward path gain

$\Delta = 1 - (\text{sum of all individual loop gains}) + (\text{sum of gain products of all possible two nontouching loops}) - (\text{sum of gain products of all possible three nontouching loops}) + \dots$

Δ_i is obtained from Δ by removing the loops which are touching the *i*th forward path.

Consider the following signal flow graph in order to understand the basic terminology involved here.



Path

It is a traversal of branches from one node to any other node in the direction of branch arrows. It should not traverse any node more than once.

Examples – $y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow y_5$ and $y_5 \rightarrow y_3 \rightarrow y_2$

Forward Path

The path that exists from the input node to the output node is known as **forward path**.

Examples – $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_6$ and $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_5 \rightarrow y_6$.

Forward Path Gain

It is obtained by calculating the product of all branch gains of the forward path.

Examples – $abcde$ is the forward path gain of $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_6$ and $abge$ is the forward path gain of $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_5 \rightarrow y_6$.

Loop

The path that starts from one node and ends at the same node is known as a loop. Hence, it is a closed path.

Examples – $y_2 \rightarrow y_3 \rightarrow y_2$ and $y_3 \rightarrow y_5 \rightarrow y_3$.

Loop Gain

It is obtained by calculating the product of all branch gains of a loop.

Examples – b_j is the loop gain of $y_2 \rightarrow y_3 \rightarrow y_2$ and g_h is the loop gain of $y_3 \rightarrow y_5 \rightarrow y_3$.

Non-touching Loops

These are the loops, which should not have any common node.

Examples – The loops, $y_2 \rightarrow y_3 \rightarrow y_2$ and $y_4 \rightarrow y_5 \rightarrow y_4$ are non-touching.

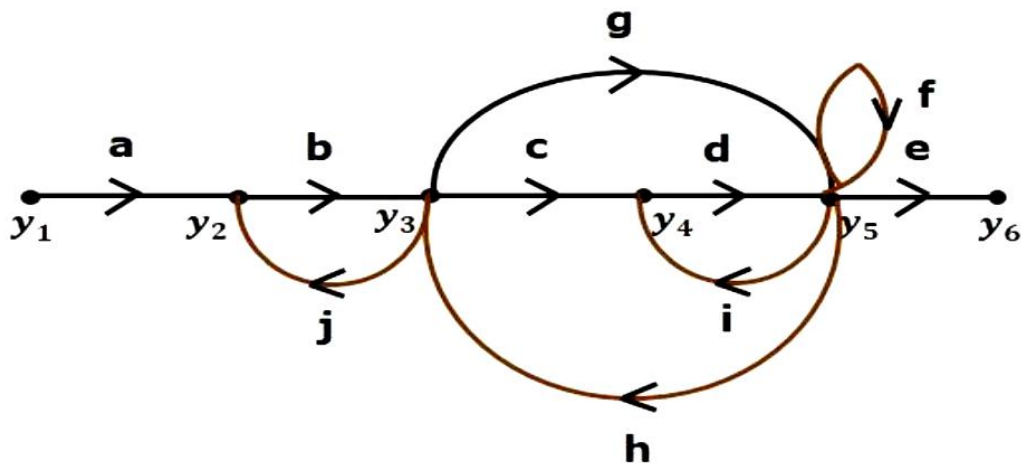
Calculation of Transfer Function using Mason's Gain Formula

Let us consider the same signal flow graph for finding transfer function.

Number of forward paths, $N = 2$.

- First forward path is - $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_6$.
- First forward path gain, $p_1 = abcde$

- Second forward path is - $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_5 \rightarrow y_6$
- Second forward path gain, $p_2 = abge$
- Number of individual loops, $L = 5$.



Loops are - $y_2 \rightarrow y_3 \rightarrow y_2$, $y_3 \rightarrow y_5 \rightarrow y_3$, $y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_3$, $y_4 \rightarrow y_5 \rightarrow y_4$ and $y_5 \rightarrow y_5$.

Loop gains are - $l_1 = bj$, $l_2 = gh$, $l_3 = cdh$, $l_4 = di$ and $l_5 = f$.

Number of two non-touching loops = 2.

- First non-touching loops pair is - $y_2 \rightarrow y_3 \rightarrow y_2$, $y_4 \rightarrow y_5 \rightarrow y_4$.
- Gain product of first non-touching loops pair $l_1 l_4 = bjdi$
- Second non-touching loops pair is - $y_2 \rightarrow y_3 \rightarrow y_2$, $y_5 \rightarrow y_5$.
- Gain product of second non-touching loops pair is $l_1 l_5 = bjf$

Higher number of (more than two) non-touching loops are not present in this signal flow graph.

We know,

$$\Delta = 1 - (\text{sum of all individual loop gains}) \\ + (\text{sum of gain products of all possible two nontouching loops}) \\ - (\text{sum of gain products of all possible three nontouching loops}) + \dots$$

Substitute the values in the above equation,

$$\Delta = 1 - (bj + gh + cdh + di + f) + (bjdi + bjf) - (0)$$

$$\Rightarrow \Delta = 1 - (bj + gh + cdh + di + f) + bjdi + bjf$$

There is no loop which is non-touching to the first forward path.

So, $\Delta_1 = 1$.

Similarly, $\Delta_2 = 1$. Since, no loop which is non-touching to the second forward path.

Substitute, $N = 2$ in Mason's gain formula

$$T = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^2 P_i \Delta_i}{\Delta}$$

$$T = \frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

Substitute all the necessary values in the above equation.

$$T = \frac{C(s)}{R(s)} = \frac{(abcde)1 + (abge)1}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

$$\Rightarrow T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

Therefore, the transfer function is -

$$T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

References:

1. I. J. Nagrath and M. Gopal, "Control Systems Engineering", New Age International Ltd.
2. B.S. Manke "Linear Control System", Khanna Publishers
3. D. Roy Choudhary, "Modern Control Engineering", PHI Learning Pvt. Ltd.