B.A./B.Sc. III Mathematics

(For Session 2019-20 and onwards)

Paper I: Analysis

Unit I

Definition and examples of metric spaces, Neighbourhoods, Interior points, Limit points, Open and closed sets, Convergent and Cauchy sequences, Completeness, Cantor's intersection theorem, Series of arbitrary terms, Convergence divergence and oscillation, Uniform convergence of sequences and series of functions, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Power series.

Unit II

Complex numbers as ordered pairs, geometric representation of complex numbers, Stereographic projection, Continuity and Differentiability of complex functions, Analytic functions, Cauchy Riemann equations, Harmonic functions.

Unit III

Complex integration, Cauchy-Goursat theorem, Cauchy's Integral formula, Formulae for first, second and nth derivatives, Cauchy's Inequality, Maximum Moduli theorem, Liouville's Theorem, Elementary functions, Mapping by elementary functions.

Unit IV

Taylor and Laurent Series, Absolute and uniform convergence of Power series, Residues and Poles, Residue theorem, Zeros and Poles of order m, Evaluation of improper real integrals, Improper Integrals and definite integrals involving sines and cosines, conformal mapping.

Reference book : Complex variables and applications by Brown & Churchill

Paper II: Linear & Abstract Algebra

Unit I

Automorphism, inner automorphism, automorphism groups and their computations, Conjugacy relations, Normaliser, Counting principle and the class equation of a finite group, Center of group of prime power order, Sylow's theorems, Sylow's p-subgroup.

Unit II

Prime and maximal ideals, Euclidean Rings, Principal ideal rings, Polynomial Rings, Polynomial over the Rational Field, The Eisenstein Criterion, Polynomial Rings over Commutative Rings, unique factorization domain.

Unit III

Vector spaces, Subspaces, Linear independence and dependence of vectors, Basis and Dimension, Quotient space, Coordinates, Computation concerning subspaces, Linear transformations, The Algebra of linear transformations, rank nullity theorem, their representation as matrices

Unit IV

Linear functionals, Dual space, transpose of a linear transformation, Characteristic values, annihilating polynomials, Cayley Hamilton Theorem, Inner product spaces, Cauchy-Schwarz inequality, orthogonal vectors, Orthogonal complements, Orthonormal sets and bases, Bessel's inequality for finite dimensional spaces, Gram-Schmidt orthogonalization process, Bilinear, Quadratic and Hermitian forms.

Reference book

1. **Topics in Algebra by I.N. Herstein.**
2. **Linear Algebra by K. Hoffman and R. Kunze.**

Paper III: Numerical Analysis

Unit I

Solution of equations: bisection, Secant, Regula Falsi, Newton Raphson's method, Newton's method for multiple roots, Newton's method for system of two non-linear equations, Interpolation, Lagrange and Hermite interpolation, Difference schemes, Divided differences, Interpolation formula using differences.

Unit II

Numerical differentiation, Numerical Quadrature: Newton Cotes Formulas, Gaussian Quadrature Formulas, System of Linear equations: Direct method for solving systems of linear equations (Gauss elimination, LU Decomposition, Cholesky Decomposition), Iterative methods (Jacobi, Gauss Seidel, Relaxation methods). The Algebraic Eigen value problem: Jacobi's method, Givens method, Power method.

Unit III

Numerical solution of Ordinary differential equations: Euler method, single step methods, Runge-Kutta method, Multi-step methods, Milne-Simpson method, Types of approximation, Least square polynomial approximation, Uniform approximation, Chebyshev polynomial approximation.

Unit IV

Difference Equations and their solutions, Shooting method and Difference equation method for solving Linear second order differential equation with boundary conditions of first, second and third type.

Reference book

1. **Numerical Methods for Engineering and scientific computation by M. K. Jain, S. R. K. Iyengar & R. K. Jain.**
2. **Introductory methods of Numerical Analysis by S. S. Sastry**

Paper IV: Differential Geometry & Tensor Analysis

Unit I

Local theory of curves- Space curves, Examples, Plane curves, tangent and normal and binormal, Osculating plane, normal plane and rectifying plane, Helices, Serret-Frenet apparatus, contact between curve and surfaces, tangent surfaces, involutes and evolutes of curves, Bertrand curves, Intrinsic equations, fundamental existence theorem for space curves, Local theory of surfaces- Parametric patches on surface curve of a surface, family of surfaces (one parameter), edge of regression, ruled surfaces, skew ruled surfaces and developable surfaces, surfaces of revolutions, Helicoids.

Unit II

Metric-first fundamental form and arc length, Local theory of surfaces (Contd.), Direction coefficients, families of curves, intrinsic properties, geodesics, canonical geodesic equations, normal properties of geodesics, geodesics curvature, geodesics polars, Gauss-Bonnet theorem, Gaussian curvature, normal curvature, Meusneir's theorem, mean curvature, Gaussian curvature, umbilic points, lines of curvature, Rodrigue's formula, Euler's theorem.

Unit III

Tensor algebra : Vector spaces, the dual spaces, tensor product of vector spaces, transformation formulae, contraction, special tensor, inner product, associated tensor.
Tensor Analysis: Contravariant and covariant vectors and tensors, Mixed tensors, Symmetric and skew-symmetric tensors, Algebra of tensors, Contraction and inner product, Quotient theorem, Reciprocal tensors, Christoffel's symbols, Covariant differentiation.

Unit IV

Gradient of scalars, Divergence of a contra-variant vector, covariant vector and conservative vectors, Laplacian of an invariant, curl of a covariant vector, irrotational vector, Riemannian

space, Riemannian curvatures and their properties, Ricci tensor, and scalar curvature, Einstein space and Einstein tensor, intrinsic derivative, Geodesics, Geodesics coordinate, parallelism of vectors.

Reference book: An introduction to Differential Geometry by T. J. Willmore